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Generalized Beal's Conjecture

Hari Kishan[1], Sarita[2] and Megha Rani[3]

[1]*Department of Mathematics, D.N. College, Meerut (U.P.)*
[2]*Department of Mathematics, DCR University, Murthal, Sonipat (Haryana)*
[3]*Department of Mathematics, RKGIT, Ghaziabad (U.P.)*

Abstract—*In this paper, the generalization of famous Fermat's Last Theorem and famous Beal's conjecture have been presented. Beal's conjecture is still an open unsolved problem having an award of one lac USD for proving or disproving the conjecture. This generalization may be helpful in the proof of the conjecture.*

1. INTRODUCTION

Pierre de Fermat in 1637 wrote in the margin of a copy of Arithmetica that no three positive integers *a, b* and *c* satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than 2. He claimed that he had a proof that was too to be fitted in the margin. This result is known as Fermat's Last Theorem or Fermat's Conjecture in the literature. Several mathematicians attempted to prove or disprove this theorem. But they could succeed partially. Finally, after 358 years of efforts by mathematicians, **Andrew Wiles** in 1994 proved this result successfully. He formally published it in 1995. This unsolved problem stimulated the development of algebraic number theory in 19th century and modularity theorem in 20th century. This theorem is among the most notable theorems in the history of mathematics and was in the Guinness Book of World Records for most difficult mathematical problems prior to its proof.

Billionaire banker **Andrew Beal** (1993) while investigating the generalization of Fermat's Last Theorem proposed a conjecture. This conjecture is known as Beal's Conjecture. It has been claimed that the same conjecture was formulated independently by Robert Tijdeman and Don Zagier. So it has also been referred as Tijdeman-Zagier conjecture.

Gandhi &Sarma (2013) attempted to disprove the Beal's conjecture.**Gola, L.W.** (2014) presented a proof of Beal's conjecture.

In this article, the generalizations of Fermat's Last Theorem and Beal's Conjecture have been presented.

2. FERMAT'S LAST THEOREM:

There exists no positive integer a, b, c for positive integer $n > 2$ such that

$$a^n + b^n = c^n. \qquad (1)$$

3. BEAL'S CONJECTURE:

If$x^l + y^m = z^n$ where x, y, z, l, m and n are positive integers with l, m and n are greater than 2 then x, y and z have a common prime factor.

Few illustrations are given below:

1. The solution $3^3 + 6^3 = 3^5$ has a common factor 3.

2. The solution $7^6 + 7^7 = 98^3$ has a common factor 7.

3. The solution $[a(a^m + b^m)]^m + [b(a^m + b^m)]^m = [(a^m + b^m)]^{m+1}$ for $a, b > 2,\ m > 2$ has a common factor $(a^m + b^m)$.

4. The solution $(a^m - 1)^{2m} + (a^m - 1)^{2m+1} = [a(a^m - 12m$, $a>1$, $m>2$ has a common factor $am-12m$.

4. GENERALIZATIONS:

Generalized Fermat's Last Theorem:

There exists no positive integer $a_1, a_2, \dots, a_m, b$ such that

$$a_1^n + a_2^n + \cdots + a_m^n = b^n, \qquad (2)$$

For$n > m$ where m and n are positive integers. For $n \le m$ there exist positive integers $a_1, a_2, \dots, a_m, b$ satisfying equation (2).

Few illustrations are given below:

1) For $m = 3$ and $n = 2$ there are positive integers 1, 2, 2 and 3 such that

$$1^2 + 2^2 + 2^2 = 3^2.$$

2) For $m = 3$ and $n = 3$ there are positive integers 3, 4, 5 and 6 such that

$3^3 + 4^3 + 5^3 = 6^3$.

3) For $m = 4$ and $n = 3$ there are positive integers 1, 1, 5, 6 and 7 such that

$1^3 + 1^3 + 5^3 + 6^3 = 7^3$.

4) For $m = 5$ and $n = 4$ there are positive integers 2, 2, 3, 4, 4 and 5 such that

$2^4 + 2^4 + 3^4 + 4^4 + 4^4 = 5^4$.

5) $30^4 + 120^4 + 272^4 + 315^4 = 353^4$ (By Norrie's (1911)

5. GENERALIZED BEAL'S CONJECTURE:

There exists no positive integer $a_1, a_2, \dots, a_m, n_1, n_2, \dots n_m, b$ such that

$a_1^{n_1} + a_2^{n_2} + \cdots + a_m^{n_m} = b^n$, for$n_1, n_2, \dots n_m > m$ unless $a_1, a_2, \dots, a_m$ and b have a common prime factor. For $n_1, n_2, \dots n_m \leq m$ the above equation may have positive integer solutions.

Few illustrations are given below:

1) The solution $36^3 + 72^3 + 108^3 = 36^4$ has a common factor 36^3. Here $m = 3$ is equal to the exponents of left hand side.
2) The solution $73^3 + 146^3 + 292^3 = 73^4$ has a common factor 73^3. Here $m = 3$ is equal to the exponents of left hand side.
3) The solution $198^3 + 297^3 + 396^3 = 99^4$ has a common factor 99^3. Here $m = 3$ is equal to the exponents of left hand side.
4) The solution $98^4 + 196^4 + 294^4 = 98^5$ has a common factor 98^4. Here $m = 3$ is less than the exponents of left hand side.
5) The solution $273^4 + 546^4 + 1092^4 = 273^5$has a common factor 273^4. Here $m = 3$ is less than the exponents of left hand side.
6) The solution $100^3 + 200^3 + 300^3 + 400^3 = 100^4$ has a common factor 100^3. Here.
$m = 4$
7) The solution $161^3 + 322^3 + 483^3 + 805^3 = 161^4$ has a common factor 161^3. Here

$m = 4$.

8) The solution $648^3 + 972^3 + 1296^3 + 1620^3 = 324^4$ has a common factor 324^3. Here $m = 4$.
9) The solution $225^3 + 450^3 + 675^3 + 900^3 + 1125^3 = 225^4$ has a common factor225^3. Here $m = 5$.
10) The solution $354^4 + 708^4 + 1062^4 + 1416^4 = 354^5$ has a common factor 354^4. Here m=4.
11) The solution $[a(a^m + b^m + c^m)]^m + [b(a^m + b^m +$ cmm+cam+bm+cmm

$= [(a^m + b^m + c^m)]^{m+1}$

for$a, b, c > 2$, $m > 2$ has a common factor$(a^m + b^m + c^m)^m$.

6. CONCLUSIONS

In this paper, the generalizations of Fermat's Last Theorem and Beal's Conjecture have been presented. Different illustrations have been presented.

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